Solitary waves and double layers in a dusty electronegative plasma

A. A. Mamun,* P. K. Shukla, and B. Eliasson†

Institut für Theoretische Physik IV, Ruhr-Universität Bochum, D-44780 Bochum, Germany

(Received 5 August 2009; revised manuscript received 13 September 2009; published 27 October 2009)

A dusty electronegative plasma containing Boltzmann electrons, Boltzmann negative ions, cold mobile positive ions, and negatively charged stationary dust has been considered. The basic features of arbitrary amplitude solitary waves (SWs) and double layers (DLs), which have been found to exist in such a dusty electronegative plasma, have been investigated by the pseudopotential method. The small amplitude limit has also been considered in order to study the small amplitude SWs and DLs analytically. It has been shown that under certain conditions, DLs do not exist, which is in good agreement with the experimental observations of Ghim and Hershkowitz [Y. Ghim (Kim) and N. Hershkowitz, Appl. Phys. Lett. **94**, 151503 (2009)].

DOI: 10.1103/PhysRevE.80.046406 PACS number(s): 52.27.Lw, 52.35.Sb, 52.35.Mw

Recently, electronegative plasmas [1–4] (plasmas with a significant amount of negative ions whose contribution cannot be neglected in any way) have attracted a great deal of attention not only because of their potential applications in microelectronic and photoelectronic industries [5] but also because of their occurrence in both laboratory devices and space environments [1–10]. The electronegative plasmas, which are observed in both laboratory devices and space, are not pure in general. They are contaminated (in most cases) by solid impurities (dust), which are not practically neutral but are charged [11] by absorbing electronegative plasma electrons and positive as well as negative ions [12–20]. Therefore, in general, electronegative plasmas are, in fact, dirty or dusty electronegative plasma [12–18,20]. On the other hand, it has been predicted by a number of authors [21–23] that negative ions in such electronegative plasmas are in Boltzmann equilibrium. This prediction has been conclusively verified by a recent laboratory experiment of Ghim and Hershkowitz [24]. Motivated by this recent laboratory experiment [24], we consider a dusty (more general) electronegative plasma containing Boltzmann electrons and Boltzmann negative ions, cold mobile positive ions, and negatively charged stationary dust, and examine the possibility for the formation of ion-acoustic (in the absence of dust) and dust-ion-acoustic [25,26] (in the presence of dust) solitary waves (SWs) and double layers (DLs) in a dusty electronegative plasma (DENP).

We consider a one-dimensional, collisionless, unmagnetized DENP composed of Boltzmann electrons, Boltzmann negative ions, cold mobile positive ions, and negatively charged stationary dust. Thus, at equilibrium we have $n_{i0}=n_{e0}+n_{n0}+z_d n_{d0}$, where n_{i0}, n_{e0}, n_{n0}, and n_{d0} are, respectively, positive ion, electron, negative ion, and dust number density at equilibrium, and z_d is the number of electrons residing onto the surface of a stationary dust. We are interested in examining the nonlinear propagation of a low phase speed (in comparison with electron and negative-ion thermal speeds), long wavelength (in comparison with $\lambda_{Dm}=(k_B T_e/4\pi n_{i0} e^2)^{1/2}$ with T_e being the electron temperature, k_B being the Boltzmann constant, and e being the magnitude of the electron charge) perturbation mode on the time scale of the ion-acoustic (IA) waves. The time scale of the IA waves is much faster than the dust plasma period so that dust can be assumed stationary. The nonlinear dynamics of the low-frequency electrostatic perturbation mode in such a DENP is described by

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i u_i)=0, \tag{1}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}=-\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=\mu_e \exp(\phi)+\mu_n \exp(\gamma\phi)-n_i+\mu_d, \tag{3}$$

where n_i is the positive-ion number density normalized by its equilibrium value n_{i0}, u_i is the positive-ion fluid speed normalized by the ion-acoustic speed $C_i=(k_B T_e/m_i)^{1/2}$, ϕ is the electrostatic wave potential normalized by $k_B T_e/e$, x is the space variable normalized by λ_{Dm}, and t is the time variable normalized by the ion plasma period $\omega_{pi}^{-1}=(m_i/4\pi n_{i0}e^2)^{1/2}$, $\mu_e=1/(1+\alpha+\beta)$, $\mu_n=\alpha/(1+\alpha+\beta)$, $\mu_d=\beta/(1+\alpha+\beta)$. $\alpha=n_{n0}/n_{e0}$, $\beta=z_d n_{d0}/n_{e0}$, $\gamma=T_e/T_n$, T_n is the negative-ion temperature, and m_i is the ion mass. The linear dispersion relation for the low phase speed (in comparison with the electron and negative-ion thermal speed) and long wavelength (in comparison with λ_{Dm}) modified IA waves [24,25] is $V_p=u_{i0}+(\mu_e+\mu_n\gamma)^{-1/2}$, where $V_p=\omega/kC_i$ is the normalized phase speed of the perturbation mode under consideration, u_{i0} is the ion drift speed [27] normalized by C_i, ω is the wave frequency, and k is the propagation constant.

To derive an energy integral [28,29] from Eqs. (1)–(3), we first make all the dependent variables depend only on a single variable $\xi=x-U_0 t$, where U_0 is the nonlinear wave speed normalized by C_i. We note that U_0 is not the Mach number, since it is normalized by C_i. If U_0 would be normalized by the phase speed (V_p) of the IA waves, it would then be called the Mach number M. So the relation between U_0 and M is $M=U_0/V_p$. Now, using the steady-state condition

*Permanent address: Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh.

†Also at Department of Physics, Umeå University, SE-901 87 Umeå, Sweden.

and imposing the appropriate boundary conditions (namely, $n_i \to 1$, $u_i \to u_{i0}$, $\phi \to 0$, and $d\phi/d\xi \to 0$ at $\xi \to \pm\infty$), one can reduce Eqs. (1)–(3) to an energy integral [28,29],

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0, \tag{4}$$

for an oscillating particle of unit mass, with pseudoposition ϕ, pseudotime ξ, and pseudopotential $V(\phi)$. The latter for our purposes reads as

$$V(\phi) = C - \mu_d\phi - \mu_e \exp(\phi) - \frac{\mu_n}{\gamma}\exp(\gamma\phi) - V_0^2\left(1 - \frac{2\phi}{V_0^2}\right)^{1/2}, \tag{5}$$

where $C=\mu_e+\mu_n/\gamma+V_0^2$ is the integration constant [which is chosen in such a way that $V(\phi)=0$ at $\phi=0$], and $V_0=U_0-u_{i0}$. It is obvious that the effect of ion drift velocity is found to shift the linear and nonlinear wave speeds only. i.e., $V_p-u_{i0}=V_p'=(\mu_e+\mu_n\gamma)^{-1/2}$ and $U_0-u_{i0}=V_0$. This means that $V_p'(V_0)$ represents the shifted linear (nonlinear) wave speed, i.e., the linear (nonlinear) wave speed relative to u_{i0}. Equations (4) and (5) are valid for arbitrary amplitude stationary nonlinear waves in our DENP. To study the possibility for the formation of the IASWs and IADLs, as well as their basic features (if they are formed), we first discuss the general conditions for the existence of the SWs and DLs. These conditions are:

(i) $V(0)=\frac{dV(\phi)}{d\phi}|_{\phi=0}=0$, which are already satisfied by the equilibrium charge neutrality condition, and by the boundary condition chosen to obtain the value of the integration constant C;

(ii) $\frac{d^2V(\phi)}{d\phi^2}|_{\phi=0}<0$, which will be satisfied if

$$V_0 > (\mu_e+\mu_n\gamma)^{-1/2} \equiv V_p'; \tag{6}$$

(iii) $V(\phi_m \neq 0)=0$, which will be satisfied if

$$C - \mu_d\phi_m - \mu_e \exp(\phi_m) - \frac{\mu_n}{\gamma}\exp(\gamma\phi_m) - V_0^2\left(1 - \frac{2\phi_m}{V_0^2}\right)^{1/2} = 0, \tag{7}$$

where ϕ_m is the amplitude of SWs or DLs; and

(iv) $\frac{dV(\phi)}{d\phi}|_{\phi=\phi_m}>0$ for positive SWs (by positive SWs we mean compressive SWs, i.e., SWs with positive potential), $\frac{dV(\phi)}{d\phi}|_{\phi=\phi_m}<0$ for negative SWs (by negative SWs we mean rarefactive SWs, i.e., SWs with negative potential), and

$$\mu_d + \mu_e\exp(\phi_m) + \mu_n\exp(\gamma\phi_m) - \left(1 - \frac{2\phi_m}{V_0^2}\right)^{-1/2} = 0 \quad \text{for DLs}. \tag{8}$$

Conditions (i)–(iii) must be satisfied for both SWs and DLs. However, in addition of these three, the first (second) of (iv) is required only for positive (negative) SWs, and the third of (iv) or Eq. (8), which represents the transition from positive SWs to negative ones or vise versa, is required for the DLs. Therefore, the minimum (critical) value of V_0 for existence of both SWs and DLs is determined by Eq. (6), the upper limit of V_0 for the existence of negative SWs (or value of V_0 for the existence of DLs) is determined by Eqs. (7) and (8), and the upper limit of V_0 for the existence of positive SWs can be determined by substituting $\phi=V_0^2/2$ into Eq. (5) (since $2\phi \leq V_0^2$ must always be valid to keep the ion number density n_i real), i.e.,

$$C + \frac{\mu_d V_0^2}{2} - \mu_e\exp\left(\frac{V_0^2}{2}\right) - \frac{\mu_n}{\gamma}\exp\left(\frac{\gamma V_0^2}{2}\right) = 0. \tag{9}$$

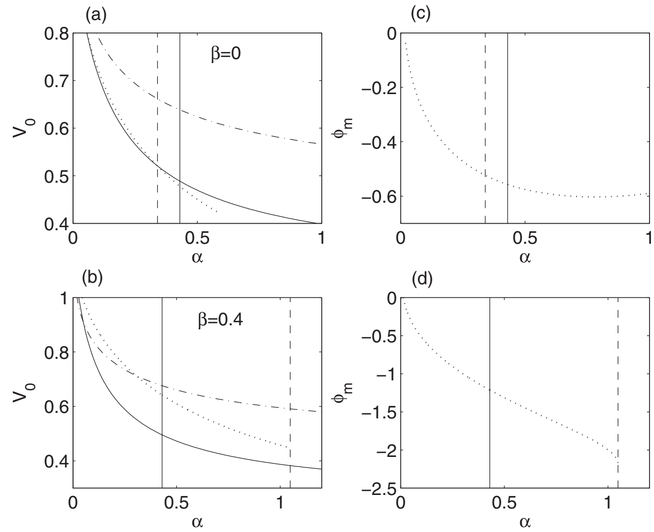

FIG. 1. Panels (a) and (b): The solid lines represent the linear wave speed V_p', the dotted lines represent the solutions of Eqs. (7) and (8), and the dashed lines represent the solutions of Eq. (9) for $\gamma=11.6$ for (a): $\beta=0$, and (b): $\beta=0.4$. Panels (c) and (d): The variation of corresponding ϕ_m with α for $\gamma=11.6$ for (c): $\beta=0$, and (d): $\beta=0.4$. The experimental value $\alpha=0.43$ of Ghim and Hershkowitz [24] is shown by the vertical solid lines. Double layers exist only to the left of the vertical dashed lines.

Now, we have numerically analyzed Eqs. (6)–(9), and find the parametric regimes (along with V_0 and ϕ_m) for the existence of IASWs and IADLs. The results of our numerical analysis are shown in Fig. 1, where panels (a) and (b) show the minimum values of V_0 for the existence of SWs or DLs (solid lines), ii) upper limit of V_o for the existence of negative SWs or value of V_0 for the existence of DLs (dotted lines), and iii) upper limit V_0 for the existence positive SWs (dashed lines) for $\gamma=11.6$ and $\beta=0$ [panel (a)], and $\beta=0.4$ [panel (b)], and panels (c) and (d) show the variation of ϕ_m with α for $\gamma=11.6$, $\beta=0$ [panel (c)], and $\beta=0.4$ [panel (d)].

Panels (a) and (b) in Fig. 1 show that (i) positive and negative SWs coexist for any point which is above the solid lines, but below both the dotted and dashed lines, (ii) only positive (negative) SWs exist for any point which is above both the solid and dotted (dashed) lines, but below the dashed (dotted) lines, and (iii) DLs exist for any point on that portion of the dotted lines which are above the solid lines. Panel (a) of Fig. 1 exhibits that for the experimental conditions of Ghim and Hershkowitz [24] DLs and negative SWs cannot be formed, but only positive IASWs can be formed. It is observed from panel (b) of Fig. 1 that the presence negatively charged ($\beta \neq 0$) is in favor of supporting DLs or of the

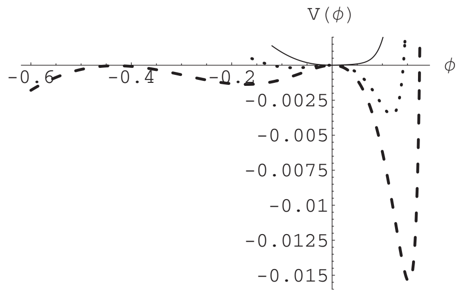

FIG. 2. Showing the coexistence of positive and negative IASWs when α=0.2, γ=15, β=0, and V_0 exceeds its critical value 0.54, i.e., V_0=0.54 (solid line), V_0=0.56 (dotted line), and V_0=0.58 (dashed line). The dashed line shows the existence of DLs. It also corresponds to the maximum value of V_0 below which negative SWs exit.

coexistence of the positive and negative SWs. Panels (c) and (d) of Fig. 1 clearly indicate that the amplitude (ϕ_m) decreases with α, but increases with β.

We now choose four different sets of plasma parameters [viz., (α=0.2, γ=15, and β=0, which correspond to the existence of IASWs for $V_0>0.54$ and of IADLs for V_0=0.5985); (α=0.43, γ=15, and β=0, which correspond to the existence of IASWs for $V_0>0.43$ and of IADLs for V_0=0.4616); (α=0.43, γ=15, and β=0.2, which correspond to IASWs for $V_0>0.49$ and IADLs for V_0=0.545); and (α=0.43, γ=15, and β=0.4, which correspond to IASWs for $V_0>0.49$ and IADLs for V_0=0.635)] and have numerically analyzed the pseudopotential $V(\phi)$ [given by Eq. (5)] in order to show the possibility for the existence of positive and negative IASWs or IADLs in our DENP. The numerical results are displayed in Figs. 3–5.

Figure 2 shows that for α=0.2, γ=15, β=0, and $0.54<V_0<0.58$ negative and positive IASWs coexist and that their amplitude increases with V_0 until V_0=0.58 at which negative IASWs do not exist but a DL (with negative potential) and a positive SW exist. Figure 3 exhibits that for α=0.43, γ=15, β=0, and $0.43<V_0<0.4616$ negative and

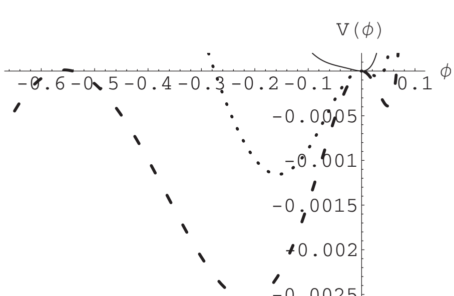

FIG. 3. Showing the coexistence of positive and negative IASWs when α=0.43, γ=15, β=0, and V_0 exceeds its critical value 0.43, i.e., V_0=0.43 (solid line), V_0=0.45 (dotted line), and V_0 =0.4616 (dashed line). The dashed line shows the existence of DLs. It also corresponds to the maximum value of V_0 below which negative SWs exit.

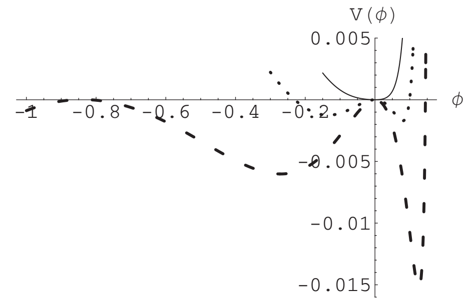

FIG. 4. Showing the coexistence of positive and negative IASWs when α=0.43, γ=15, β=0.2, and V_0 exceeds its critical value 0.43, i.e., V_0=0.43 (solid line), V_0=0.45 (dotted line), and V_0=0.545 (dashed line). The dashed line shows existence of the IADLs with negative potential. It also corresponds the maximum value of V_0 below which the negative IASWs exit.

positive IASWs coexist and that their amplitude increases with V_0 until V_0=0.4616 at which negative IASWs do not exist but a DL (with a negative potential) and a positive SW exist. Figure 4 shows that for α=0.43, γ=15, β=0.2, and $0.43<V_0<0.545$ negative and positive IASWs coexist and that their amplitude increases with V_0 until V_0=0.545 at which negative IASWs do not exist but a DL (with a negative potential) and a positive SW exist. Figure 5 shows that for α=0.43, γ=15, β=0.4, and $0.49<V_0<0.58$ negative and positive IASWs coexist and that their amplitude increases with V_0 until V_0=0.58. It also shows that at V_0=0.635 neither positive nor negative SWs exist, but DLs (with negative potential) exist.

Figures 2–5 can also provide a visualization of the amplitude (ϕ_m) and the width ($|\phi_m|/|V_m|$) of the IASWs or IADLs, where ϕ_m is the maximum (in case of positive SWs or DLs) or minimum (in case of negative SWs or DLs) value of ϕ satisfying $V(\phi)$=0, i.e., the value of ϕ for which the curves in Figs. 2–5 cross the positive (in case of positive SWs or DLs) or negative (in case of negative SWs or DLs) ϕ-axis,

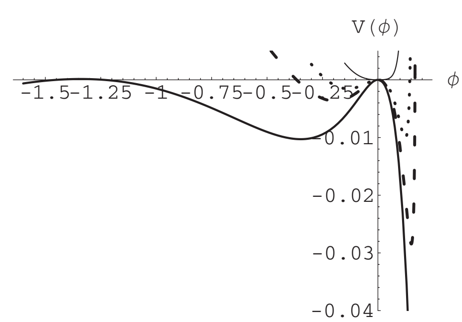

FIG. 5. Showing the coexistence of positive and negative IASWs when α=0.43, γ=15, β=0.4, and V_0 exceeds its critical value 0.49, i.e., V_0=0.49 (upper solid line), V_0=0.55 (dotted line), V_0=0.58 (dashed line), and V_0=0.635 (lower solid line). The dashed line shows the maximum value of V_0 above which positive SWs do not exit. The lower solid line shows the existence of DLs, as well as the maximum value of V_0 below which the negative SWs exit.

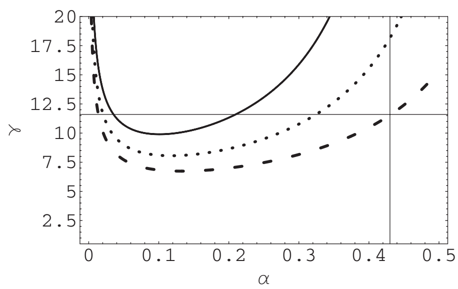

FIG. 6. $C_3(V_0=V_p')=0$ curves: the boundaries separating the parametric regimes for the existence of the positive and negative IASWs and IADLs for different values of β, viz., $\beta=0$ (solid line), $\beta=0.2$ (dotted line), and $\beta=0.4$ (dashed line). The values of α and γ corresponding to the experimental condition of Ghim and Hershkowitz [24] are shown by the horizontal and vertical lines.

and $|V_m|$ is the minimum value of $V(\phi)$ in the potential wells formed in the positive (in case of positive SWs or DLs) or negative (in case of negative SWs or DLs) ϕ axis.

To compare the results obtained from numerical analysis (presented above) with those obtained from some analytic analysis, we now study the basic features of small amplitude IASWs and IADLs. Let us first consider small amplitude IASWs by expressing Eqs. (5) and (7) as

$$V(\phi) = C_2\phi^2 + C_3\phi^3, \tag{10}$$

$$C_2 + C_3\phi_m = 0, \tag{11}$$

where

$$C_2 = \frac{1}{2}\left(\frac{1}{V_0^2} - \mu_e - \mu_n\gamma\right), \tag{12}$$

$$C_3 = \frac{1}{6}\left(\frac{3}{V_0^4} - \mu_e - \mu_n\gamma^2\right). \tag{13}$$

The parameters C_2 and C_3 [Eqs. (12) and (13)] determine the existence of solitary waves. We note that that condition (i) is already satisfied [which is obvious from Eq. (10)], condition (ii) is satisfied if $C_2<0$

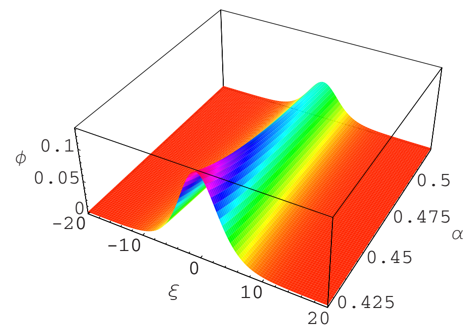

FIG. 8. (Color online) Solitary potential structures for the parametric regime below the solid curve of Fig. 6, i.e., $\beta=0$, $\gamma=15$, $V_0=V_p'+0.01$, and $\alpha=0.4$–0.5.

which reduce to Eq. (6). So, using Eqs. (10) and (11) in Eq. (4), we can find small amplitude solitary wave solutions of Eq. (4) as

$$\phi = \phi_m\,\mathrm{sech}^2\left(\frac{\xi}{\Delta}\right), \tag{14}$$

where $\phi_m=-C_2/C_3$ and $\Delta=\sqrt{-2/C_2}$. This clearly indicates that IASW solutions of Eq. (4) exist if and only if $C_2<0$, i.e., $V_0>\sqrt{(1+\alpha+\beta)/(1+\alpha\gamma)}\equiv V_p'$, and that the compressive (rarefactive) IASWs exist if $C_3>0(C_3<0)$. Thus, $C_3(V_0=V_p')=0$ will give the boundaries separating the parametric regimes for the existence of the positive and negative IASWs. These boundaries for different values of β are shown in Fig. 6.

To examine the basic features (amplitude and width) of the IASWs represented by Eq. (14), we have shown how the solitary potential profiles change with α and β. These are displayed in Figs. 7–9.

The amplitude of the IASWs becomes ∞ for any set of plasma parameters corresponding to any point on $C_3=0$ curves shown in Fig. 6. This means that the theory for the small amplitude IASWs is not valid at any point around $C_3=0$ curves. It is obvious from Figs. 6–9 that for any set of the plasma parameters (α,β,γ) corresponding to a point below (above) $C_3=0$ curves, we have positive (negative) IASWs, and that as this point (α,β,γ) moves down (up)

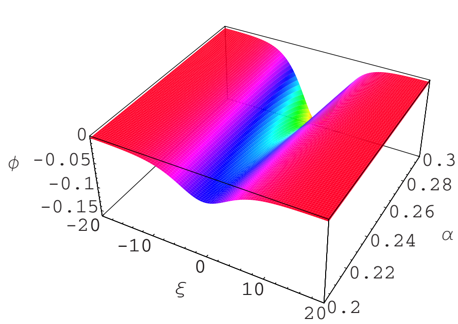

FIG. 7. (Color online) Solitary potential structures for the parametric regime above the solid curve of Fig. 6, i.e., $\beta=0$, $\gamma=15$, $V_0=V_p'+0.01$, and $\alpha=0.2$–0.3.

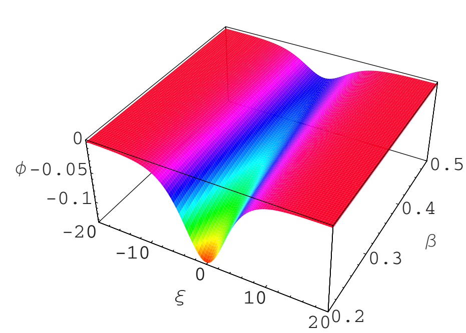

FIG. 9. (Color online) Solitary potential structures for the parametric regime above the dashed curve of Fig. 6, i.e., $\alpha=0.4$, $\gamma=15$, $V_0=V_p'+0.01$, and $\beta=0.2$–0.5.

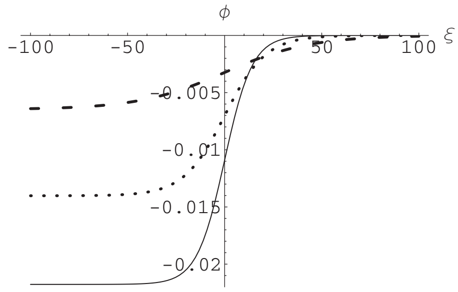

FIG. 10. DL structures for $\beta=0$, $\gamma=15$, and $\alpha=0.20$ along with corresponding $V_0=0.549$ (solid curve), $\alpha=0.23$ along with corresponding $V_0=0.526$ (dotted curve), and $\alpha=0.26$ along with corresponding $V_0=0.507$ (dashed curve).

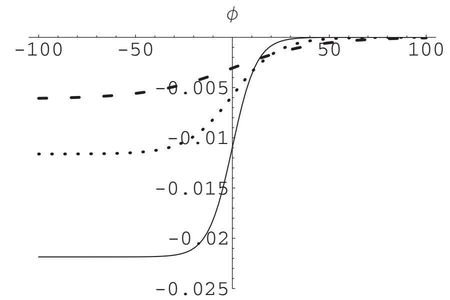

FIG. 12. DL structures for $\beta=0.4$, $\gamma=15$, and $\alpha=0.40$ along with corresponding $V_0=0.513$ (solid curve), $\alpha=0.43$ along with corresponding $V_0=0.496$ (dotted curve), and $\alpha=0.46$ along with corresponding $V_0=0.507$ (dashed curve).

from the $C_3=0$ curves, the amplitude of the positive (negative) IASWs decreases.

We now focus on small but finite amplitude IADLs by expressing Eqs. (5), (7), and (8) as

$$V(\phi) = C_2\phi^2 + C_3\phi^3 + C_4\phi^4, \tag{15}$$

$$C_2 + C_3\phi_m + C_4\phi_m^2 = 0, \tag{16}$$

$$2C_2 + 3C_3\phi_m + 4C_4\phi_m^2 = 0, \tag{17}$$

where ϕ_m is the double layer height, and

$$C_4 = \frac{1}{24}\left(\frac{15}{8V_0^6} - \mu_e - \mu_n\gamma^3\right). \tag{18}$$

We can combine the conditions Eqs. (16) and (17), and obtain

$$\phi_m^2 = \frac{C_2}{C_4} = \frac{C_3^2}{4C_4^2}, \tag{19}$$

where C_2, C_3, and C_4 are given by Eqs. (12), (13), and (18), respectively. Now, using Eqs. (15) and (19) in Eq. (4), we can find a weak double layer solution of Eq. (4) as

$$\phi = \frac{\phi_m}{2}\left[1 - \tanh\left(\frac{\xi}{\delta}\right)\right], \tag{20}$$

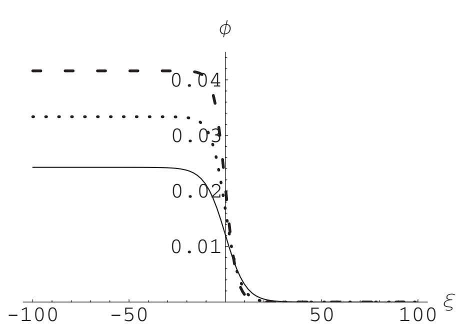

FIG. 11. DL structures for $\beta=0$, $\gamma=15$, and $\alpha=0.40$ along with corresponding $V_0=0.449$ (solid curve), $\alpha=0.45$ along with corresponding $V_0=0.435$ (dotted curve), and $\alpha=0.50$ along with corresponding $V_0=0.424$ (dashed curve).

where $\delta=\sqrt{-8C_4/C_3^2}$. It clearly indicates that small amplitude DLs [Eq. (20)] can only be formed if $C_4<0$, and that small amplitude positive (negative) double layers will be formed if $C_3>0$ ($C_3<0$). Thus, $C_3(V_0=V_p')=0$ will give the boundaries separating the parametric regimes for the existence of the positive and negative IADLs. These boundaries for different values of β are shown in Fig. 6. We have also analyzed the value of V_0 along with corresponding values of γ, α and β for which doubler layers can be formed, i.e., for which $C_3^2=4C_2C_4$ is satisfied. We have found that for small amplitude limit our analysis completely agrees with dotted curves in the left panel of Fig. 1. To make it more clear, we have taken few sets of γ, α, β, and V_0 satisfying the double layer condition ($C_3^2=4C_2C_4$), and have shown how the DL potential profiles change with α and β. These are depicted in Figs. 10–12. We note that the amplitude of the IADLs vanishes for any set of plasma parameters corresponding to any point on $C_3=0$ curves shown in Fig. 6. It is obvious from Figs. 6 and 10–12 that for any set of the plasma parameters (α,β,γ) corresponding to a point below (above) $C_3=0$ curves, we have positive (negative) IADLs, and that as this point (α,β,γ) moves down (up) from the $C_3=0$ curves, the amplitude of the positive (negative) IADLs increases.

To summarize, we have considered a DENP containing Boltzmann electrons, Boltzmann negative ions, cold mobile positive ions, and negatively charged stationary dust, and have examined the possibility for the formation of IASWs and IADLs in such a DENP. We have used the pseudopotential approach which is valid for arbitrary amplitude IASWs and IADLs. We have found here that the electronegative plasma under consideration supports the coexistence of positive and negative SWs as well as DLs for a suitable parametric regime, and that the presence of negatively charged stationary dust significantly modify the basic features of these electrostatic SWs and DLs. We have also shown here that under certain conditions [24], DLs do not exist, which is in good agreement with the experimental observations of Ghim and Hershkowitz [24].

A. A. Mamun acknowledges support from the Alexander von Humboldt Stiftung (Bonn, Germany). B. Eliasson acknowledges the support from the Swedish Research Council (VR).

[1] D. Vender, W. W. Stoffels, E. Stoffels, G. M. W. Kroesen, and F. J. de Hoog, Phys. Rev. E **51**, 2436 (1995).
[2] T. Kimura, K. Imagaki, and K. Ohe, J. Phys. D **31**, 2295 (1998).
[3] S. V. Berezhnoj *et al.*, Appl. Phys. Lett. **77**, 800 (2000).
[4] R. N. Franklin, Plasma Sources Sci. Technol. **11**, A31 (2002).
[5] M. A. Lieberman and A. Lichtenberg, *Principle of Plasma Discharges and Materials Processing*, 2nd ed. (Wiley, New York, 2005).
[6] P. Chabert, A. J. Lichtenberg, and M. A. Lieberman, Phys. Plasmas **14**, 093502 (2007).
[7] N. Plihon, P. Chabert, and C. S. Corr, Phys. Plasmas **14**, 013506 (2007).
[8] A. Meige *et al.*, Phys. Plasmas **14**, 053508 (2007).
[9] A. J. Coates *et al.*, Geophys. Res. Lett. **34**, L22103 (2007).
[10] T. H. Chung, Phys. Plasmas **16**, 063503 (2009).
[11] P. K. Shukla and B. Eliasson, Rev. Mod. Phys. **81**, 25 (2009).
[12] A. A. Mamun and P. K. Shukla, Phys. Plasmas **10**, 1518 (2003).
[13] N. D'Angelo, J. Phys. D **37**, 860 (2004).
[14] B. M. Annaratone, T. Antonova, H. M. Thomas, and G. E. Morfill, Phys. Rev. Lett. **93**, 185001 (2004).
[15] B. M. Annaratone and J. E. Allen, J. Phys. D **38**, 26 (2005).
[16] S. H. Kim and R. L. Merlino, Phys. Plasmas **13**, 052118 (2006).
[17] R. L. Merlino and S. H. Kim, Appl. Phys. Lett. **89**, 091501 (2006).
[18] M. Rosenberg and R. L. Merlino, Planet. Space Sci. **55**, 1464 (2007).
[19] M. Rosenberg and R. L. Merlino, J. Plasma Phys. **75**, 495 (2009).
[20] A. A. Mamun, R. A. Cairns, and P. K. Shukla, Phys. Lett. A **373**, 2355 (2009).
[21] A. J. Lichtenberg *et al.*, Plasma Sources Sci. Technol. **6**, 437 (1997).
[22] R. N. Franklin and J. Snell, J. Plasma Phys. **64**, 131 (2000).
[23] E. A. Bogdanov and A. A. Kudryavtsev, Tech. Phys. Lett. **27**, 905 (2001).
[24] Y. Ghim (Kim) and N. Hershkowitz, Appl. Phys. Lett. **94**, 151503 (2009).
[25] P. K. Shukla and V. P. Silin, Phys. Scr. **45**, 508 (1992).
[26] A. Barkan, N. D'Angelo, and R. L. Merlino, Planet. Space Sci. **44**, 239 (1996).
[27] I. G. Kouznetsov, A. J. Lichtenberg, and M. A. Lieberman, J. Appl. Phys. **86**, 4142 (1999).
[28] I. B. Bernstein, J. M. Greene, and M. D. Kruskal, Phys. Rev. **108**, 546 (1957).
[29] R. Z. Sagdeev, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1966), Vol. 4, p. 23.